SOLVING WORD PROBLEMS USING TWO EQUATIONS

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The strategies used to solve problems using two equations are:

- **Step 1: Represent one of the unknowns as *x* and the other unknown as *y*.**
- **Step 2: Translate the information about the variables into two equations using the two unknowns.**
- **Step 3: Solve the system of equations for *x* and *y*.**

Example # 1:

One number is 8 more than another number and the sum of the two numbers is 26. Find the numbers.

Solution:

Strategy: let x = the smaller number

y = the larger number

Since one number is 8 more than the other number, the first equation is

$$y = x + 8$$

Example # 1:

One number is 8 more than another number and the sum of the two numbers is 26. Find the numbers.

Solve the system:

$$y = x + 8$$

$$x + y = 26$$

Substitute the value for *y* in the second equation and solve for *x* since $y = x + 8$

$$x + y = 26$$

$$x + x + 8 = 26$$

$$2x + 8 = 26$$

$$2x + 8 - 8 = 26 - 8$$

$$2x = 18$$

$$\frac{2x}{2} = \frac{18}{2}$$

$$x = 9$$

Example # 1:

One number is 8 more than another number and the sum of the two numbers is 26. Find the numbers.

find the other number:

$$y = x + 8$$

$$y = 9 + 8$$

$$y = 17$$

hence, the numbers are *9* and *17*.

check the second equation.

$$x + y = 26$$

$$9 + 17 = 26$$

$$26 = 26$$

Example # 2:

The sum of the digits of a two-digit number is 15. If the digits are reversed, the new number is 9 more than the original number. Find the number.

Solution:

Strategy: **Let x = the ten's digit**

y = the one's digit

$10x + y$ = original number

$10y + x$ = new number with digits reversed

Example # 2:

The sum of the digits of a two-digit number is 15. If the digits are reversed, the new number is 9 more than the original number. Find the number.

Since the sum of the digits of the number is 15, the first equation is

$$x + y = 15$$

Since reversing the digits gives a ne number which is 9 more than the original number, the equation is

$$(10x + y) + 9 = (10y + x)$$

Solve the system:

$$x + y = 15$$

$$10x + y + 9 = 10y + x$$

Example # 2:

The sum of the digits of a two-digit number is 15. If the digits are reversed, the new number is 9 more than the original number. Find the number.

Solve the first equation for y, substitute in the second equation and find x.

$$x + y = 15$$

$$x - x + y = 15$$

$$y = 15 - x$$

$$10x + y + 9 = 10y + x$$

$$10x + (15 - x) + 9 = 10(15 - x) + x$$

$$10x + 15 - x + 9 = 150 - 10x + x$$

$$9x + 24 = 150 - 9x$$

$$9x + 9x + 24 = 150 - 9x + 9x$$

$$18x = 150 - 24$$

$$18x = 126$$

$$\frac{18x}{18} = \frac{128}{18}$$

$$x = 7$$

Example # 2:

The sum of the digits of a two-digit number is 15. If the digits are reversed, the new number is 9 more than the original number. Find the number.

Find *y*:

$$x + y = 15$$

$$7 + y = 15$$

$$y = 15 - 7$$

$$y = 8$$

Hence the number is 78.

Check the information in the second equation.

Original number is 78

Reversed number is 87

Since 87 is 9 more than 78, the answer is correct.

EXAMPLE # 3:

A person has 8 coins consisting of quarters and dimes. If the total amount of this change is $1.25, how many of each kind of coin are there?

Solution:

Strategy: let x = the number of quarters

Let y = the number of dimes

$25x$ = the value of the quarters

And $10y$ = the value of the dimes

Since there are 8 coins, the first equation is

$$x + y = 8$$

Since the total values of the quarters plus the dimes is $1.25, the second equation is

$$25x + 10y = 125$$

Solve the system:

$$x + y = 8$$

$$25x + 10y = 125$$

EXAMPLE # 3:

A person has 8 coins consisting of quarters and dimes. If the total amount of this change is $1.25, how many of each kind of coin are there?

Find the value of y in the first equation. substitute it in the second equation and solve for x.

$$x + y = 8$$

$$x - x + y = 8 - x$$

$$y = 8 - x$$

$$25x + 10y = 125$$

$$25x + 10(8 - x) = 125$$

$$25x + 80 - 10x = 125$$

$$15x + 80 = 125$$

$$15x + 80 - 80 = 125 - 80$$

$$15x = 45$$

$$\frac{15x}{15} = \frac{45}{15}$$

$$x = 3$$

Example # 3:

A person has 8 coins consisting of quarters and dimes. If the total amount of this change is $1.25, how many of each kind of coin are there?

Find *y*:

$$x + y = 8$$

$$3 + y = 8$$

$$3 - 3 + y = 8 - 3$$

$$y = 5$$

Hence, there are 3 quarters and 5 dimes.

Check if their sum is $1.25.

3 quarters = *3 x $0.25 = $0.75*

5 dimes = *5 x $0.10 = $0.50*

$0.75 + $0.50 = $1.25

Example # 4:

A merchant mixes some coffee costing $4 a pound with some coffee costing $3 a pound. How much of each must be used in order to make 20 pounds of mixture costing $3.75 a pound.

Solution:

Strategy:

Let x = the amount of $4 coffee used

y = the amount of $3 coffee used

Since the total amount of the mixture is 20 pounds, the first equation is

$$x + y = 20$$

Since the cost of the mixture is $3.75, the second equation is

$$4x + 3y = 20(3.75)$$

Example # 4:

A merchant mixes some coffee costing $4 a pound with some coffee costing $3 a pound. How much of each must be used in order to make 20 pounds of mixture costing $3.75 a pound.

Solve the system:

$$x + y = 20$$

$$4x + 3y = 20(3.75)$$

Solve the first equation for x. Substitute in the second equation and solve for y.

$$x + y = 20$$

$$x + y - y = 20 - y$$

$$x = 20 - y$$

Example # 4:

A merchant mixes some coffee costing $4 a pound with some coffee costing $3 a pound. How much of each must be used in order to make 20 pounds of mixture costing $3.75 a pound.

Substitute:

$$4x + 3y = 20$$

$$4(20 - y) + 3y = 20(3.75)$$

$$80 - 4y + 3y = 75$$

$$80 - y = 75$$

$$80 - 80 - y = 75 - 80$$

$$-y = -5$$

$$\frac{-Y}{-1} = \frac{-5}{-1}$$

$$Y = 5 \text{ pounds}$$

Solve for x:

$$x + y = 20$$

$$x + 5 = 20$$

$$x + 5 - 5 = 20 - 5$$

$$x = 15 \text{ pounds}$$

Example # 4:

A merchant mixes some coffee costing $4 a pound with some coffee costing $3 a pound. How much of each must be used in order to make 20 pounds of mixture costing $3.75 a pound.

Hence, 15 pounds of the $4 coffee are needed and 5 pounds of of the $3 coffee are needed

Check the second equation.

$$4x + 3y = 20(3.75)$$

$$4(15) + 3(5) = 75$$

$$60 + 15 = 75$$

$$75 = 75$$

Use two equations with two unknowns.

1. One number is 4 times another number. If their sum is 40, find the numbers.

2. The sum of the digits of a two-digit number is 14. If the digits are reversed, the new number is 18 more than the original number. Find the number.

3. A person has 18 coins, some of which are nickels and the rest of which are dimes. If the total amount of the coins is $1.30, find the number of nickels and dimes.

4. Matt is 4 times older than mike. In 10 years, he will be twice as old as mike. Find their ages.